Malaysian Journal of Mathematical Sciences 2(2): 55-60 (2008)

The Solution of the One Species Lotka-Volterra Equation Using Variational Iteration Method

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ABSTRACT

This paper will present a numerical comparison between the variational iteration method (VIM) and the Adomian decomposition method (ADM) for solving the one species Lotka-Volterra equation. The numerical results demonstrate that the VIM are quite accurate and readily implemented.

Keywords: Variational iteration method (VIM), Adomian decomposition method (ADM), Lotka-Volterra equations

INTRODUCTION

The Lotka-Volterra equations model the dynamic behavior of an arbitrary number of competitors [1]. Though originally formulated to describe the time history of a biological system, these equations find their application in a number of engineering fields such as simultaneous chemical and nonlinear control. In fact, the one predator one prey Lotka-Volterra model is one of the most popular ones to demonstrate a simple nonlinear control system.

The accurate solution of the Lotka-Volterra equations may become a difficult task either if the equations are stiff (even with a small number of species), or when the number of species is large [2].

The Adomian decomposition method, was proposed by Adomian [3] initially with the aims to solve frontier physical problem. It has been applied to a wide class of deterministic and stochastic problems, linear and nonlinear, in physics, biology and chemical reactions etc. For nonlinear models, the method has shown reliable results in supplying analytical approximation that converges very rapidly [4].

In recent years, a great deal of attention has been devoted to study the VIM given by Professor Ji-Huan He for solving a wide range of problems whose mathematical models yield differential equation or system of differential equations [5–13].

The purpose of this paper is to make a numerical comparison between VIM and ADM for solving the one species Lotka-Volterra equation.

He's variational iteration method

This method, which is a modified general Lagrange's multiplier method [14], has been shown to solve effectively, easily and accurately a large class of nonlinear problems [5–12]. The main feature of the method is that the solution of a mathematical problem with linearization assumption is used as initial approximation or trial function, then a more highly precise approximation at some special point can be obtained. This approximation converges rapidly to an accurate solution. To illustrate the basic concepts of the VIM, we consider the following nonlinear differential equation:

$$Lu + Nu = g(x); \qquad (1)$$

where L is a linear operator, N is a nonlinear operator, and $g(x)$ is an inhomogeneous term. According to the VIM [5–8], we can construct a correction functional as follows:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda (Lu_n(\xi) + N\tilde{u}_n(\xi) - g(\xi))d\xi \qquad (2)$$

where¸ is a general Lagrangian multiplier [14], which can be identified optimally via the variational theory, the subscript n denotes the n^{th}-order approximation, $\tilde{u}_n$ is considered as a restricted variation [5–8], i.e. $\delta\tilde{u}_n = 0$.

Analysis of one species Lotka-Volterra equations

The one species competing for a given finite source of food:

$$\frac{dN_i}{dt} = N_i(b_i + \sum_{j=1}^{m} a_{ij}N_j), i = 1,2,...,m. \qquad (3)$$

where a and b are constants. This equation has an exact solution:

$$\mathrm{N(t)} = \frac{\mathrm{be}^{\mathrm{bt}}}{\frac{\mathrm{b+aN(0)}}{\mathrm{N(0)}} - ae^{bt}} \quad \text{for } b \neq 0, \tag{5}$$

$$N(t) = \frac{N(0)}{1 - aN(0)t} \quad \text{for } b = 0. \tag{6}$$

To solve equation (3) by the standard Adomian decomposition method (ADM) [3] with initial condition $N(0) = 0.1$, using the inverse operator $L^{-1}(\cdot) = \int_0^t (\cdot)dt$ we get:

$$N_0 = N(0), N_{n+1}(t) = \int_0^t (bN_n + aA_{1,n})dt, n \geq 0, \tag{7}$$

where the Adomian polynomial:

$$A_{1,n} = \sum_{k=0}^{n} N_k N_{n-k}. \tag{8}$$

To solve equation (3) by means of He's variational iteration method, we construct a correction function:

$$N_{n+1}(t) = N_n(t) + \int_0^t \lambda(s)\left[\frac{dN_n(s)}{ds} - bN_n(s) - a\tilde{N}_n^2(s)\right]ds, \tag{9}$$

where $\tilde{N}_n$ is considered as restricted variations, which mean $\delta\tilde{N}_n = 0$. Its stationary conditions can be obtained as follows:

$$1 + \lambda(t) = 0, \tag{10}$$

$$\lambda'(s) + b\lambda(s)\big|_{s=t} = 0. \tag{11}$$

The Lagrange multipliers, therefore, can be identified as ¸ $\lambda(s) = -e^{-b(s-t)}$ and the following variational iteration formula can be obtained by:

$$N_{n+1}(t) = N_n(t) - \int_0^t \frac{e^{-bs}}{e^{-bt}} \left[\frac{dN_n(s)}{ds} - bN_n(s) - aN_n^2(s) \right] ds. \qquad (12)$$

We can take the linearized solution $N(t) = Ce^{bt}$ as the initial approximation $N(0)$, then we get:

$$N_1(t) = Ce^{bt} + \frac{aC^2 e^{bt}(e^{bt} - 1)}{b}. \qquad (13)$$

The condition $N(0) = 0.1$ gives us $C = 0.1$. Thus:

$$N_1(t) = 0.1e^{bt} + \frac{a0.01\ e^{bt}(e^{bt} - 1)}{b}. \qquad (14)$$

In the same manner, the rest of the components of the iteration formulae (12) can be obtained using the Maple Package.

Numerical results and discussion

We now obtain numerical solutions of one species Lotka-Volterra equations. We compare with the exact solution , ADM, VIM. Table 1 shows the case of one species comparison between the 2-iterate of VIM, 3-term in the series of ADM and exact solution for $b = 1$; $a = -3$ and $N(0) = 0.1$.

TABLE 1: Numerical comparison when $b = 1$, $a = -3$, and $N(0) = 0.1$.

t	Exact solution	ADM, ϕ_3	2-Iterate VIM
0.0	0.1000000000	0.1000000000	0.1000000000
0.1	0.1071367895	0.1071400000	0.1071389974
0.2	0.1145329055	0.1145600000	0.1145545368
0.3	0.1221638511	0.1222600000	0.1222530752
0.4	0.1300011386	0.1302400000	0.1302589889
0.5	0.1380126120	0.1385000000	0.1386248288
0.6	0.1461628997	0.1470400000	0.1474445088
0.7	0.1544139905	0.1558600000	0.1568693714
0.8	0.1627259130	0.1649600000	0.1671263185
0.9	0.1710574954	0.1743400000	0.1785357990
1.0	0.1793671754	0.1840000000	0.1915249307

CONCLUSION

In this paper, we compare between the exact solution, VIM and ADM applied to one species Lotka-Volterra equations. Comparisons with the Adomian decomposition method (ADM) show that the VIM is a powerful method for the solution of nonlinear equations. The advantage of the VIM over the ADM is that there is no need for the evaluation of the Adomian polynomials.

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